

18. Prove that if K is a field and if $\sigma_1, \dots, \sigma_n$ are distinct automorphisms of K then it is impossible to find elements $a_1, \dots, a_n$, not all 0, in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + \dots + a_n\sigma_n(u) = 0$ for all $u \in K$.
19. Prove that If A is an algebra, with unit element, over F , then A is isomorphic to a subalgebra of $A(V)$ for some vector space V over F .
20. State and prove Frobenius theorem.

APRIL/MAY 2023

GMA21/DMA21 — ALGEBRA – II

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.



1. Define field.
2. Give the example of extension field.
3. Define simple extension.
4. Define multiplicity.
5. Define Galois group.
6. Write symmetric polynomial.
7. Define normal extension.
8. Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.
9. Define algebraic over a field.
10. Define norm.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that There is an isomorphism τ^{**} of $F[x]/(f(x))$ onto $F[t]/(f'(t))$ with the property that for every $\alpha \in F$, $\alpha\tau^{**} = \alpha'$.

Or

- (b) Prove that the mapping $\psi: F[x] \rightarrow F(\alpha)$ defined by $h(x)\psi = h(\alpha)$ is homomorphism.
12. (a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial (that is, of positive degree) common factor.

Or

- (b) Prove that if F is a field of characteristic $p \neq 0$, then the polynomial $x^{p^n} - x \in F[x]$, for $n \geq 1$ has distinct roots.
13. (a) Prove that the fixed field of G is a subfield of K .

Or

- (b) Prove that $G(K, F)$ is a subgroup of the group of all automorphisms of K .

14. (a) Prove that G is solvable if and only if $G^{(k)} = (e)$ for some integer k .

Or

- (b) Prove that Let A be an algebra, with unit element, over F , and suppose that A is of dimension m over F . Then every element in A satisfies some non-trivial polynomial in $F[x]$ of degree at most m .
15. (a) Prove that Let C be the field of complex numbers and suppose that the division ring D is algebraic over C . Then $D = C$.

Or

- (b) Prove that if $\alpha \in H$ then $\alpha^{-1} \in H$ if and only if $N(\alpha) = 1$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that if α is constructible then α lies in some extension of the rationals of degree a power of 2.
17. Prove that if $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F , then there is an extension E of F , such that $[E:F] = n$, in which $p(x)$ has a root.